MODELING INSTRUCTION

SAMPLES OF MECHANICS UNIT 2: CONSTANT VELOCITY

CONTENTS

This sample is designed to provide prospective users with enough material to evaluate the content and flow of the unit, but omits many worksheets/readings/etc, as well as the quizzes and tests. The Teacher Notes contain a complete list of all activities and documents for the unit.

For more information, visit our website at www.modelinginstruction.org

Unit 2: Particle Moving with Constant Velocity

Overview of Schober's Updates, 2010

The worksheets have been slightly expanded, not so much by adding more questions, but instead by making sure multiple representations have been included in every problem. More background has been given to distinguishing between velocity and speed by emphasizing the difference between displacement and "odometer reading." Though "odometer reading" is not a standard physics term, it seems enormously helpful in clarifying the concepts. Worksheet 4 is expanded to include a relative motion question that sets up the fundamental conundrum underlying special relativity. I want my students to feel comfortable enough with a chunk of physics that they can see beyond our studies and appreciate the vast realm of knowledge they could now pursue.

Editing-in-Process Comments

This set of modeling materials is the result of a decade of incremental adjustments the Modeling Materials to improve them for the best impact on my students. Beginning from the 1998 version of the modeling materials, my edits have incorporated my ideas and experiences as well as materials from others who have graciously shared their ideas with me.

Though the student materials are thoroughly tested, the teacher notes are still incomplete. I have added some material to the teacher's notes, but without any refining. In particular, a "sequence" section has been added that corresponds to my version of the materials, and sporadic additions to the instructional notes explain my changes. It will be necessary to convene a team of editors to make a number of decisions regarding incorporating revisions into the "standard" modeling materials before the teacher notes can be fully fleshed out.

I am frequently asked for answer keys. The keys I have made are handwritten and are not in a shareable form. In addition to answers, effective keys for the modeling materials should include questions to ask during whiteboarding, rationales for the questions, and misconceptions to be aware of – and this is a project that has not yet been tackled.

Also included are the tests and quizzes I use, some of which vary significantly from the original materials. I request that you keep these assessments secure so that they remain valid and useful.

Feel free to contact me with questions,

Mark Schober
jmschober@gmail.com

Unit 2: Particle Moving with Constant Velocity

Instructional goals

1. Reference frame, position and trajectory
   Choose origin and positive direction for a system
   Define motion relative to frame of reference
   Distinguish between vectorial and scalar concepts
   (displacement vs distance, velocity vs speed)

2. Particle Model
   Kinematical properties (position and velocity) and laws of motion
   Derive the following relationships from position vs time graphs

   $\Delta x = x_f - x_0$

   $\bar{v} = \dfrac{\Delta x}{\Delta t}$

   $x = \bar{v}t + x_0$

   $\Delta x = \bar{v}t$

3. Multiple representations of behavior
   Introduce use of motion map and vectors
   Relate graphical, algebraic and diagrammatic representations.

4. Dimensions and units
   Use appropriate units for kinematical properties
   Dimensional analysis

5. Software
   Intro to *Conceptual Kinematic Tutorial* (PAS)

Sequence

1. Buggy Motion Lab

2. Reading: Motion Maps

3. Lab: Multiple Representations of Motion: Ultrasonic Motion Detector Lab; discuss lab

4. Worksheet 1: Motion Maps and Position vs. Time graphs

5. Worksheet 2: Motion Maps and Velocity vs. Time graphs

6. Quiz 1: Quantitative Motion maps

7. Constant Velocity Lab Practicum: Dueling Buggies

8. Worksheet 3: Position vs. time graphs and velocity vs. time graphs

9. Quiz 2: Average speed

10. Worksheet 4: Velocity vs. time graphs and displacement

11. Worksheet 5: Multiple representations of motion

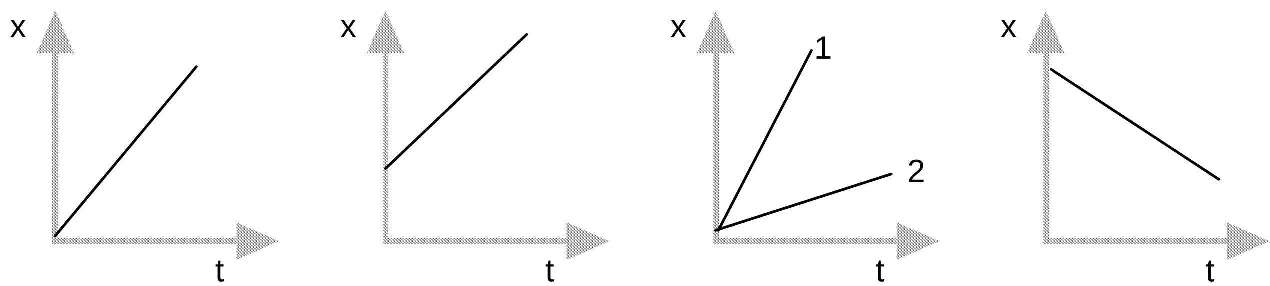
12. Review Sheet

13. Constant Velocity Test

Overview

1. It is important to describe motion in terms of *position* and time, rather than distance. Position is much less ambiguous than distance (sometimes regarded as the path length, sometimes as displacement). Some authors use 's' to describe this variable; we prefer 'x' for horizontal motion (and 'y' when the motion is vertical). We advise against the use of 'd'. When it comes time to discuss the slope of the position-time graph, the definition for velocity, $v = \dfrac{\Delta x}{\Delta t}$, naturally arises. Change in position is superior to change in distance; the latter is a difference of differences. Change in position is the definition of *displacement,* the quantity that helps distinguish velocity from speed. Displacement can be (+) or (-), distance is by definition (+).

2. When discussing the meaning of the graphs, be sure to use a wide variety of examples.

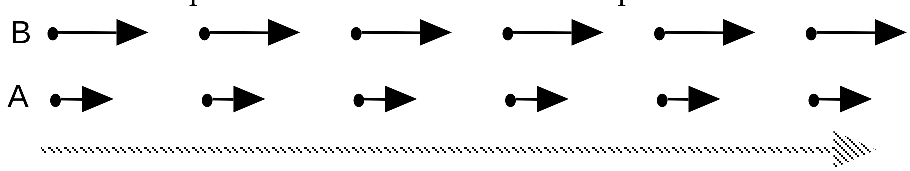


   Induce the students to describe the motion in full detail (e.g., the object starts somewhere to the right of the origin and moves to the left at constant speed).

3. Using an enlargement of one of their graphs, have the students manually calculate the slope and compare to the value obtained by GA. Students have been conditioned to think of slope only as "rise over run" or Δy over Δx.

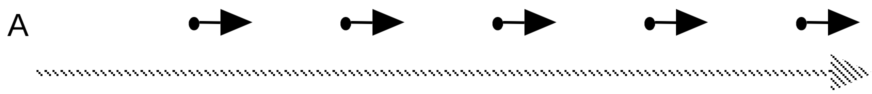
4. Make sure that they have a thorough grasp of the relationship between slope and velocity. The answer "1's slope is greater than 2's" is not a guarantee of understanding. It would be profitable to have students model the behavior of the object represented by a variety of graphs. If you have an ultrasonic motion detector, this is great fun!

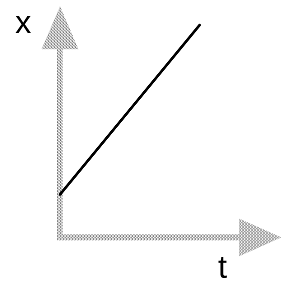
5. Work on making motion maps to represent the position-time behavior of moving objects. Make sure that these semi-quantitative devices are faithful representations.



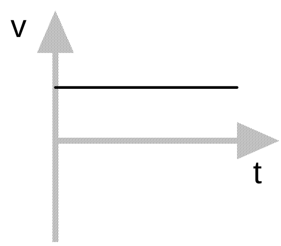
   Students have been known to draw the motion map above and state that B was moving faster than A because the velocity vectors were longer.

6. Make sure that students can, given an algebraic statement, an **x** vs **t** graph, or a motion map, recreate the other two representations.

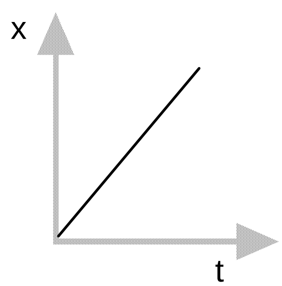
Given: 

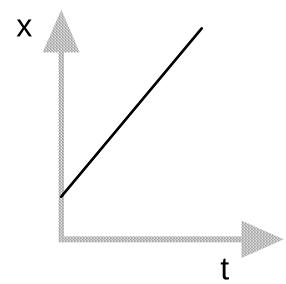
They should be able to write: $x = vt + x_0$ and draw 

7. Be sure to make the connection between **x** vs **t** graphs and **v** vs **t** graphs. "Stacking" the curves helps to illustrate this relationship.

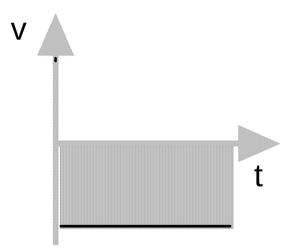


Make the point that the **v** vs **t** graph yields no information about starting point. The **v** vs **t** graph at left could represent either of the **x** vs **t** graphs below.





8. Make the point that the area under a **v** vs **t** graph represents the displacement, Δx of the object. This could be both (+) and (-). Avoid always using the trivial case.



Instructional notes

1. Battery-powered vehicle lab

Apparatus

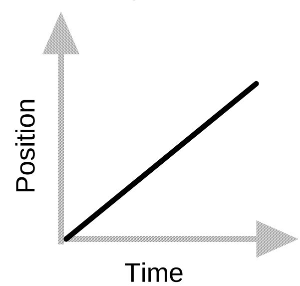
- any slow-moving battery powered toy vehicle
- stop watches
- meter sticks
- masking tape
- Graphical Analysis

Pre-lab discussion

- Let the vehicle move across table and ask for observations. List observations and then ask which items are quantifiable. Lead them to observe that the tractor moves at constant speed; i.e., that it travels equal distances in equal time intervals.
- The dependent variable is position (x). Emphasize that we are dealing with position, not displacement or distance traveled.
- The independent variable is time (t). Emphasize time as a *clock reading* and not an interval of time. (Why make time independent? Because when time is graphed on the horizontal axis the slope will be equivalent to velocity.)

Lab performance notes

- Stopwatches and battery-powered vehicles are easier to use than "stomper" cars and photogates. (Honors classes may be able to handle use of photogates at this stage.)
- However you choose to have the students collect the data, they should be reminded to perform multiple trials with at least 6 data pairs/trial. Averaging the values of position helps them develop a sense of the precision they should carry through the analysis. Otherwise they are guilty of adhering to Lillenthal's Laws:
  1- If reproducibility is a problem, conduct only 1 test.
  2- If a straight line plot is required, collect only two data points.



Post-lab discussion

- Focus discussion on the position versus time relationship.
- Use slope-intercept form to write equation of line (e.g. $x = (0.85\frac{m}{s})t + 0.12m$).
- Discuss the slope of the line as being a constant. Introduce the label units of slope (m/s).
- Identify v (velocity) as the slope in the slope-intercept equation.
- Discuss the vertical intercept and the "5% rule-of-thumb". In most cases, the intercept is negligible.
- From specific equation, write general mathematical model $\Delta x = \bar{v}t$. Discuss displacement (Δx) when initial position is not zero.

2. Reading: Motion Maps

3. Lab: Multiple Representations of Motion: Ultrasonic Motion Detector Lab

4. Worksheet 1: Motion Maps and Position vs. Time graphs

I've added robust, multi-representational constant velocity questions to the beginning of the worksheet, and ranking questions that require students to discriminate between displacement and odometer reading at the end.

5. Worksheet 2: Motion Maps and Velocity vs. Time graphs

Motion maps are requested on the first page, and questions at the end reinforce the interpretation of area under a velocity-time graph.

6. Quiz 1: Quantitative Motion maps

7. Constant Velocity Lab Practicum: Dueling Buggies

Students measure properties of a fast and slow buggy so that they can predict where the two buggies will meet when driven toward each other from starting lines provided by the teacher.

I break the class into groups of 6 to 8 students. Each group gets a few minutes to measure everything they think would be important to know about the motion of two battery-powered buggies, one fast and one slow. I make slow buggies by wrapping one battery in electrical tape and using a strip of aluminum foil as a shunt across it. After the students have made the measurements they deem necessary, I take each group's buggies away from them and mark two start lines, one for the fast buggy and one for the slow buggy. The students need to figure out and then mark where the two buggies will meet. Once they have made their prediction, I return the group's original buggies back to them and they test their prediction.

Depending on how much time you devote to the activity, you can ask to solve the problem multiple ways. I let them use any techniques they want. Once the have a solution, I ask them to solve it again with another technique to check their solution. Possible techniques might include: life-size motion map, graphing to find the intersection, two equations and two unknowns, and ratio reasoning. The larger group allows subgroups to solve the problem multiple ways at the same time.

I don't usually grade the activity, but precision points could be awarded. Points can also be awarded for the group's explanation of the solution. All students are asked to be prepared to explain their group's solution. A random draw chooses the student to present the solution for the group, so the group is responsible for making sure every member of the group understands how they reached their solution.

My lab tables can be written on with chalk for marking starting and meeting lines, but spreading whiteboards across lab tables or the floor would accomplish the same thing. With an initial separation of 3.5 meters, my students seldom miss the intersection by more than a few centimeters.

8. Worksheet 3: Position vs. time graphs and velocity vs. time graphs

Ranking questions added to emphasize the difference between avg. velocity and speed

9. Quiz 2: Average speed

10. Worksheet 4: Velocity vs. Time Graphs and Displacement

Original question #1 was confusing, asking quantitative questions from qualitative information -- now all parts are quantitative
Questions have been added to the second situation
Relative motion questions added which set up a peek at the motivation for special relativity

11. Worksheet 5: Multiple representations of motion

12. Review Sheet

13. Constant Velocity Test

Particle Moving with Constant Velocity Model

Key ideas:

By the end of this unit, you should be able to do the following:

1. You should be able to determine the average velocity of an object in two ways:
    - determining the slope of an x vs. t graph
    - using the equation
2. You should be able to determine the displacement of an object in two ways:
    - finding the area under a v vs. t graph
    - using the equation
3. Given an x vs. t graph:
    - describe the motion of the object (starting position, direction of motion, velocity)
    - draw the corresponding v vs. t graph
    - draw a motion map for the object
    - determine the average velocity of the object (slope)
    - write the mathematical model which describes the motion
4. Given a v vs. t graph:
    - describe the motion of the object (direction of motion, how fast)
    - draw the corresponding x vs. t graph
    - determine the displacement of the object (area under curve).
    - draw a motion map for the object
    - write a mathematical model to describe the motion

Terms and Definitions:

Average Velocity = Slope of x-t graph $= \dfrac{\text{Change in Position}}{\text{Change in Time}} = \dfrac{x_f - x_i}{t_f - t_i}$

Δx = change in position = xf - xi = displacement = area under v-t graph. Displacement is the straight-line distance between the starting point and the ending point. Displacement also reports the direction of motion.

Odometer reading = total distance traveled along a path to get from the starting position to the ending position.

Average Speed = Distance along path / Change in Time

Scalar = A Quantity that tells "how much" only i.e. speed, time, mass, odometer reading . . .

Vector = A quantity that tells *how much* and *which direction* i.e. velocity, displacement, force . . .

Qualitative = Conceptually correct, but not numerically precise

Quantitative = Numerically Accurate

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Constant Velocity Particle Model Ultrasonic Motion Detector Lab:

Multiple Representations of Motion

Do the following for each of the situations below:

a. Move, relative to the motion detector, so that you produce a position vs. time graph that closely approximates the graph shown.

b. In the space provided, describe how you must move in order to produce the position vs. time graph shown in the space to the right of the velocity vs. time graph. Be sure to include each of the following in your description: starting position, direction moved, type of motion, relative speed.

c. On the velocity vs. time axes, sketch the velocity vs. time graph that corresponds to the position vs. time graph shown.

d. In the space provided, sketch the motion map that corresponds to the motion described in the position vs. time graph.

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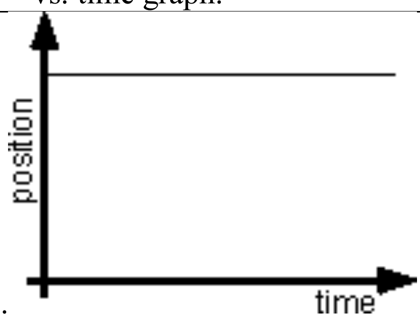

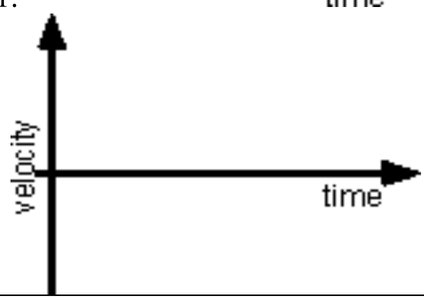

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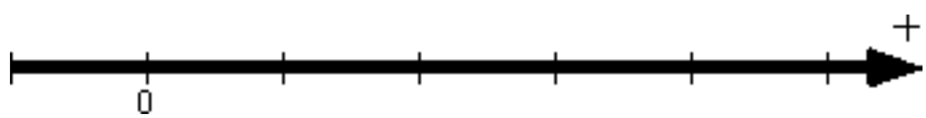

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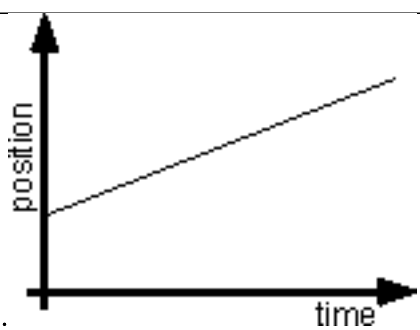

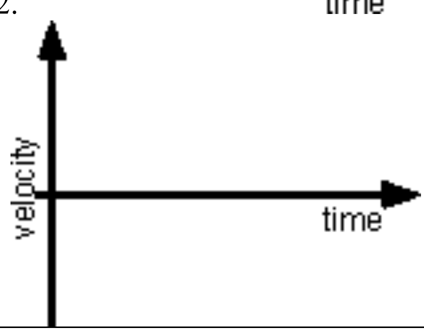

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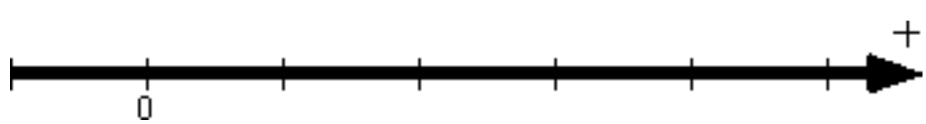

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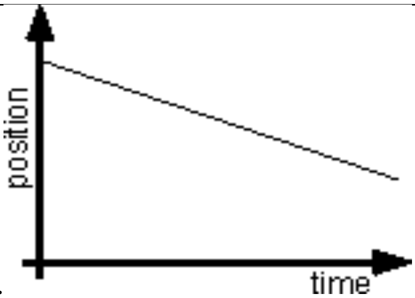

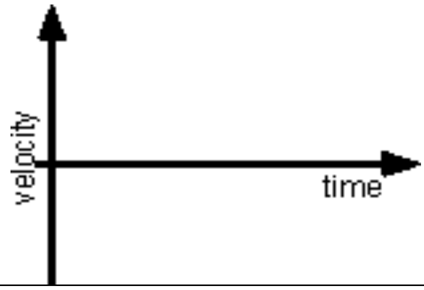

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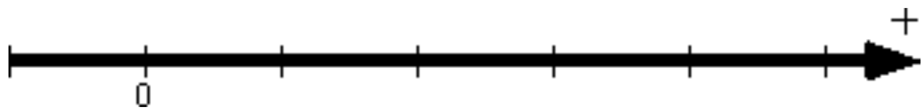

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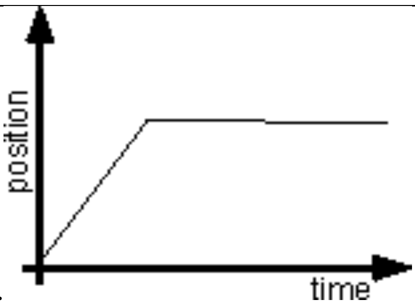

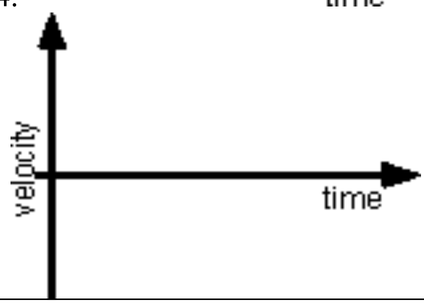

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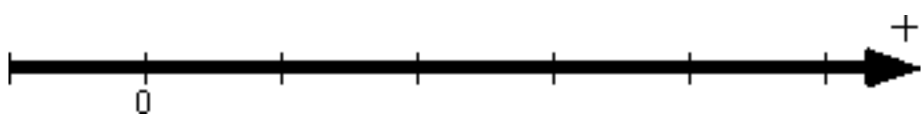

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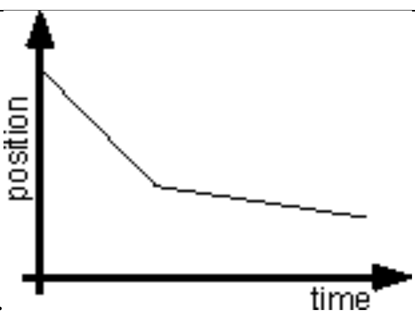

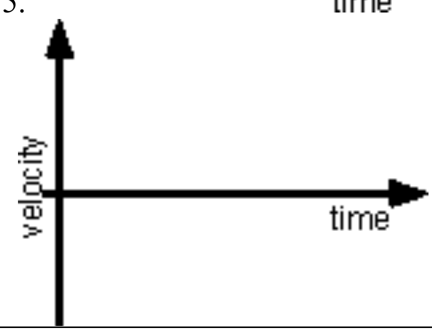

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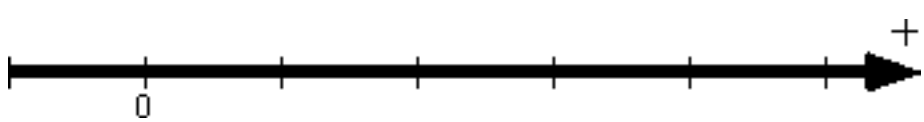

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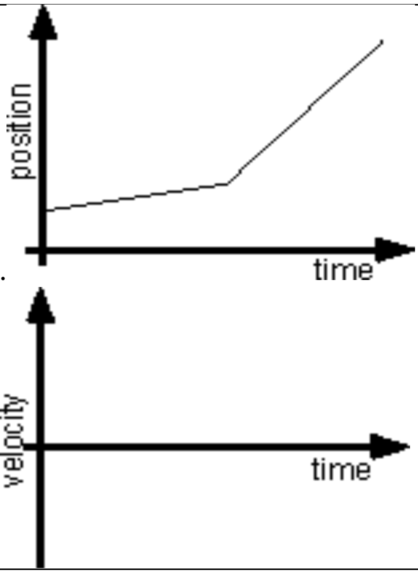

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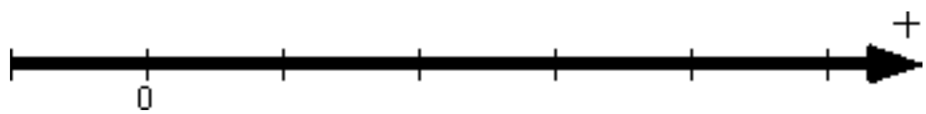

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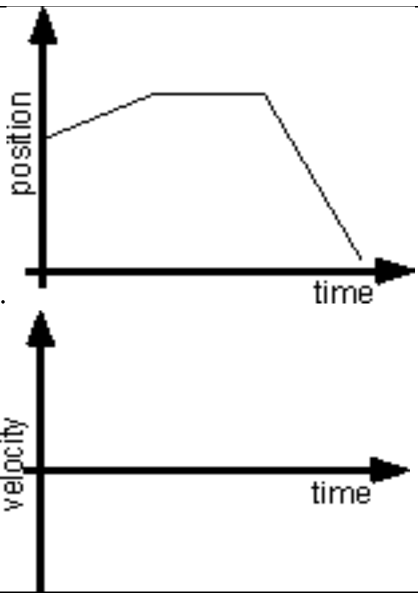

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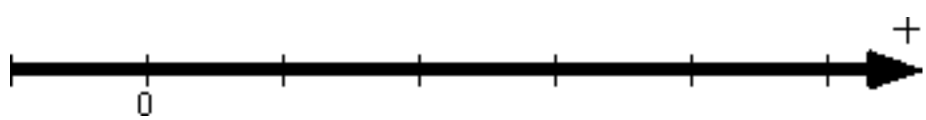

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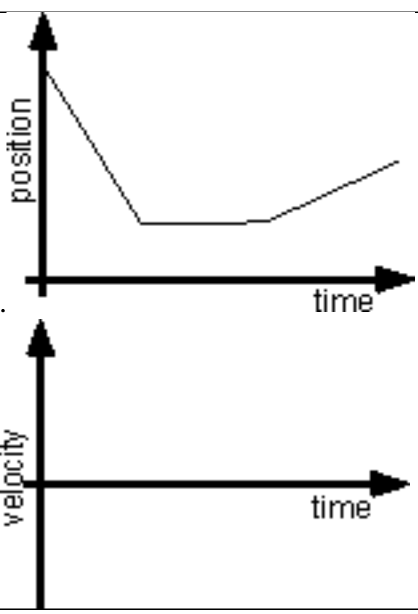

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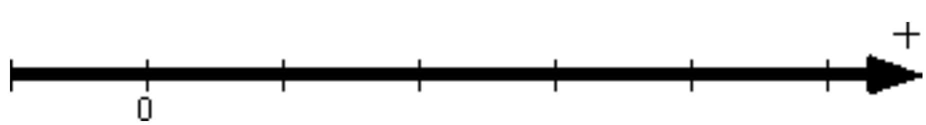

For the following, match the given velocity-time graph.

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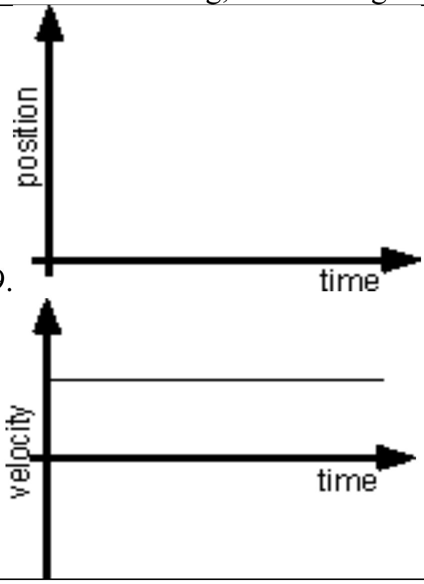

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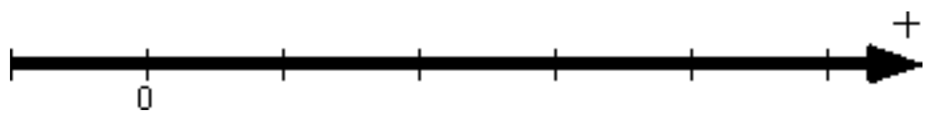

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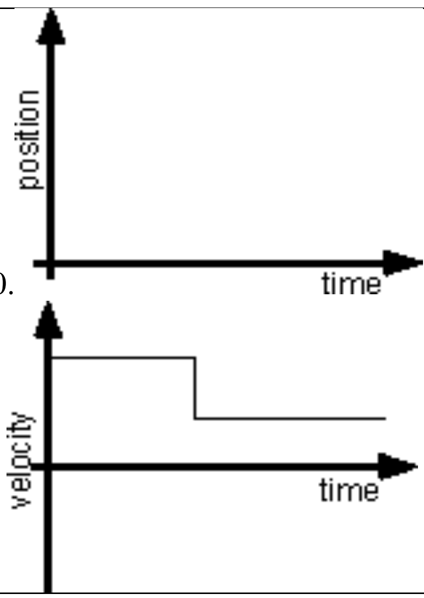

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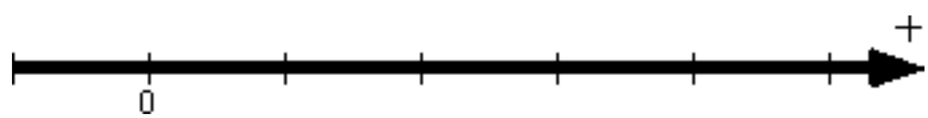

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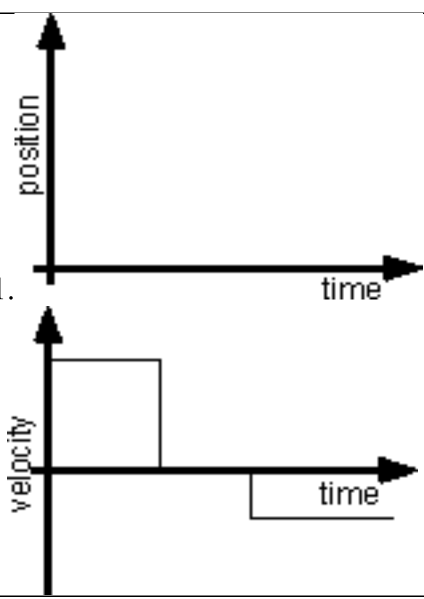

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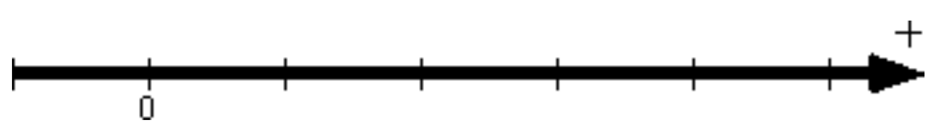

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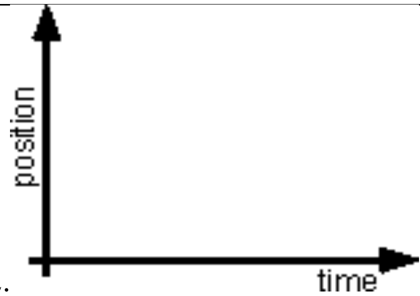

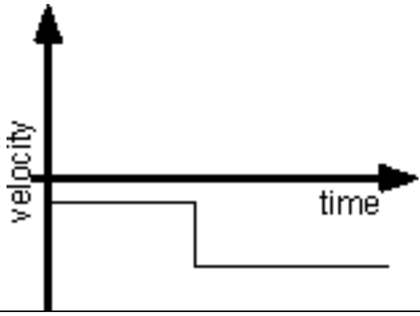

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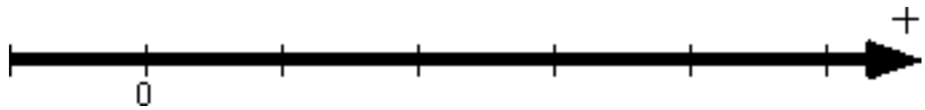

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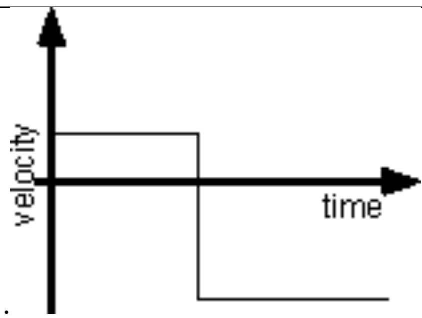

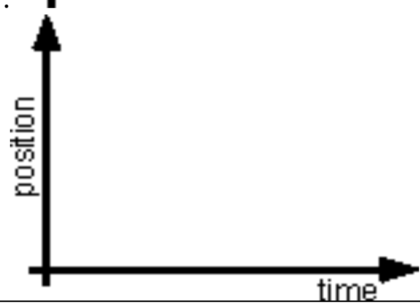

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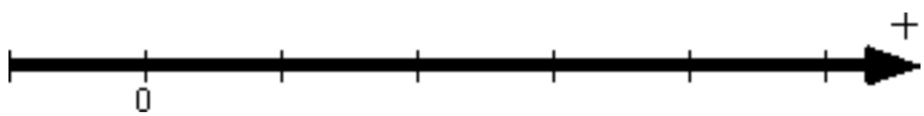

Name_______________________________________

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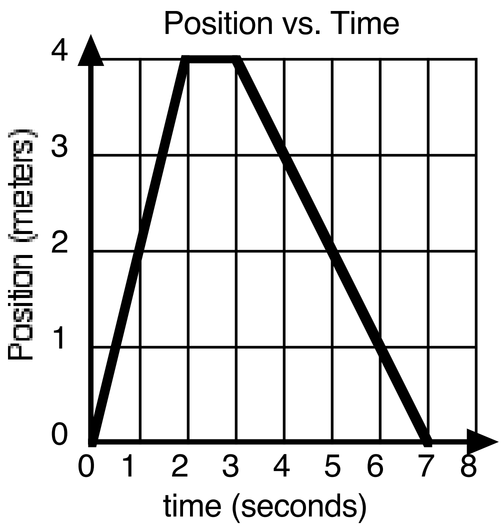
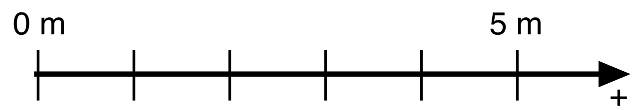
Constant Velocity Particle Model Worksheet 1:

Motion Maps and Position vs. Time Graphs

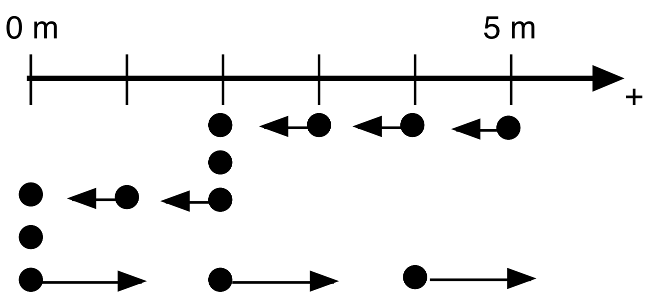
1. Given the following position vs. time graph, draw a motion map with one dot for each second.

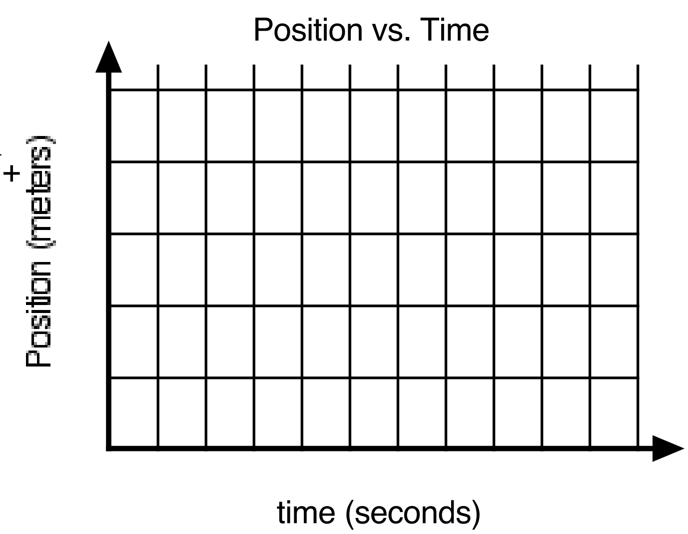




Describe the motion of the object in words:

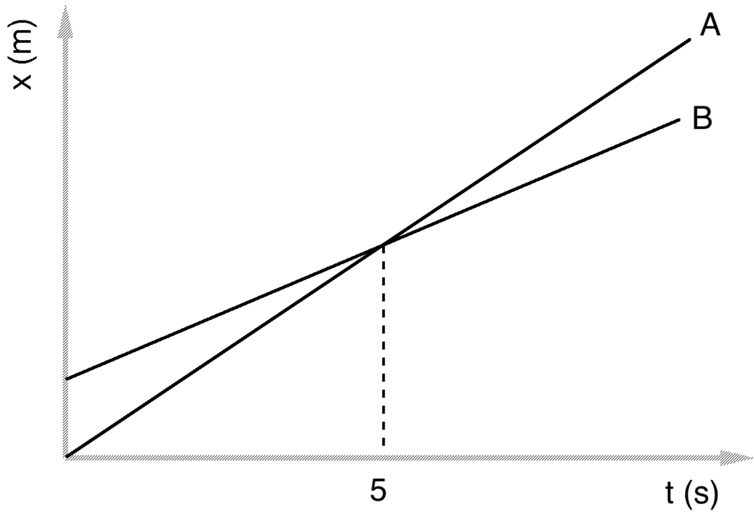
2. Given the following motion map, where positions have been recorded with one dot each second, draw a position vs. time graph.





Describe the motion of the object in words:

3. Consider the position vs. time graph below for cyclists A and B.



a. Do the cyclists start at the same point? How do you know? If not, which is ahead?

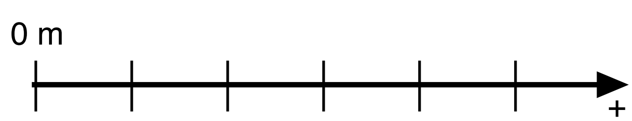
b. At t= 7s, which cyclist is ahead? How do you know?

c. Which cyclist is traveling faster at 3s? How do you know?

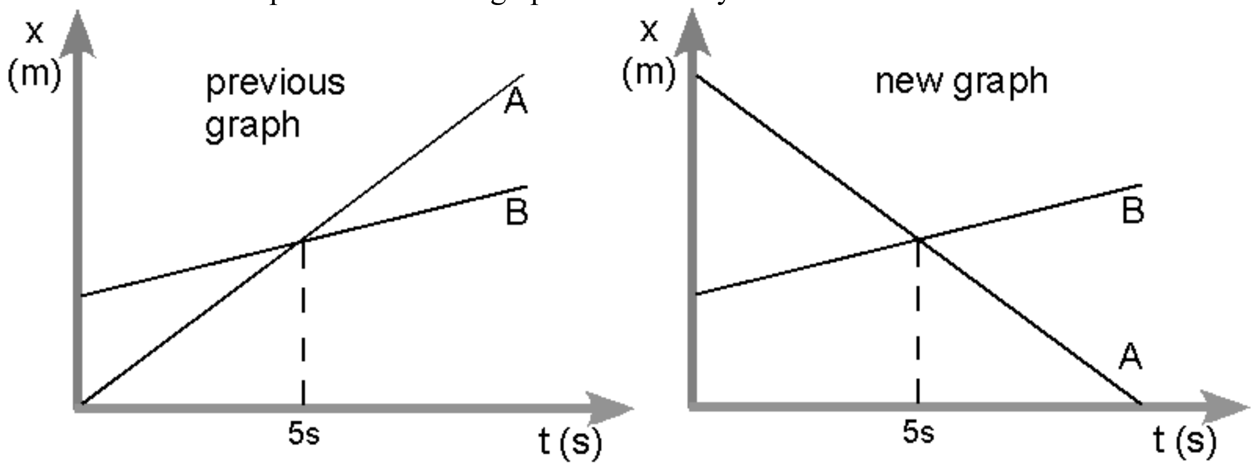
d. Are their velocities equal at any time? How do you know?

e. What is happening at the intersection of lines A and B?

f. Draw a motion map for cyclists A and B.



4. Consider the new position vs. time graph below for cyclists A and B.



a. How does the motion of the cyclist A in this graph compare to that of A in question 3?

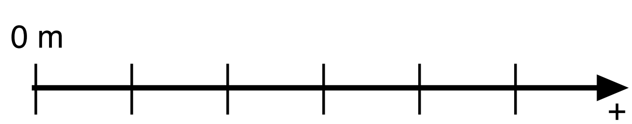
b. How does the motion of cyclist B in this graph compare to that of B in question 3?

c. Which cyclist has the greater speed? How do you know?

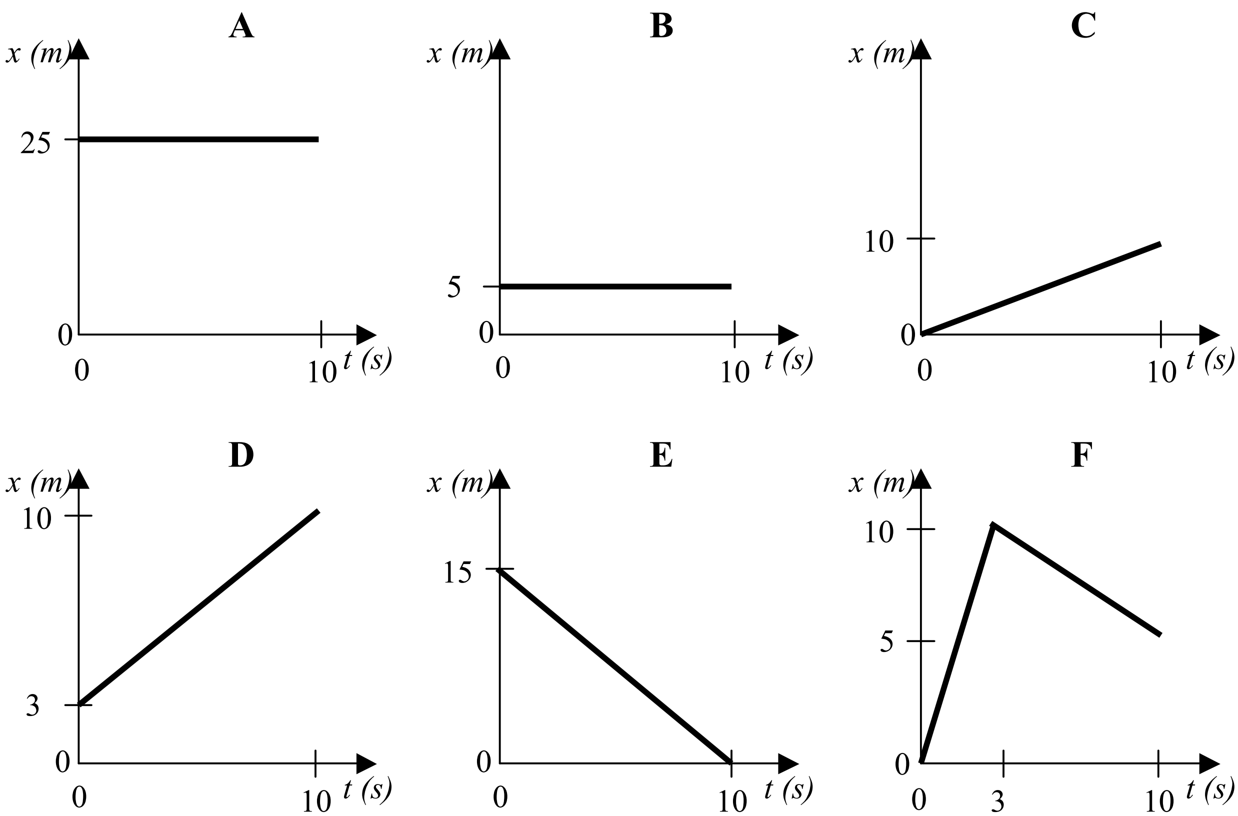
d. Describe what is happening at the intersection of lines A and B.

e. Which cyclist has traveled further during the first 5 seconds? How do you know?

f. Draw a motion map for cyclists A and B.



5. To rank the following, you may need to look at the key ideas sheet for the difference between *displacement* and *odometer reading*.



a. Rank the graphs according to which show the greatest **displacement** from the beginning to the end of the motion.

Most positive → 1______ 2______ 3______ 4______ 5______ 6______ ← Most negative

Explain your reasoning for your ranking:

b. Rank the graphs according to which show the greatest **odometer reading** from the beginning to the end of the motion.

Greatest 1________ 2________ 3________ 4________ 5________ 6________ Least

Explain your reasoning for your ranking:

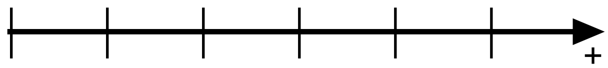
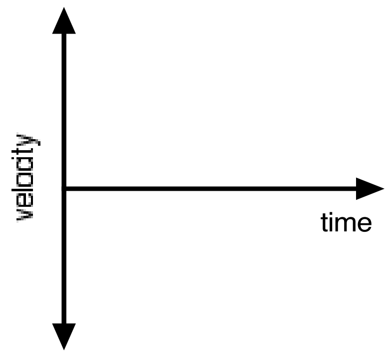
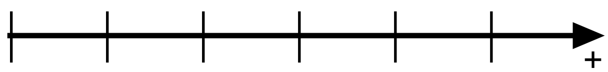
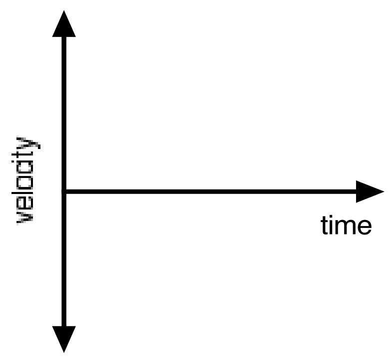
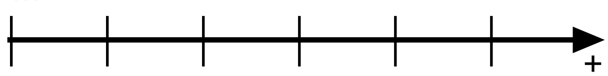
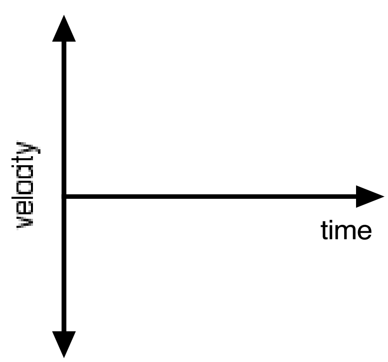
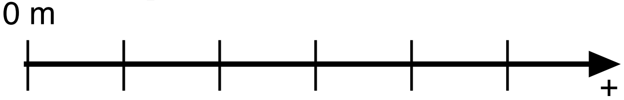
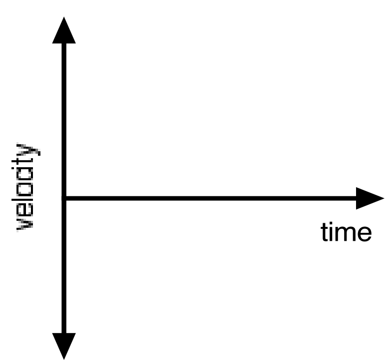
Name____

Date__________ Pd____

Constant Velocity Particle Model Worksheet 2:

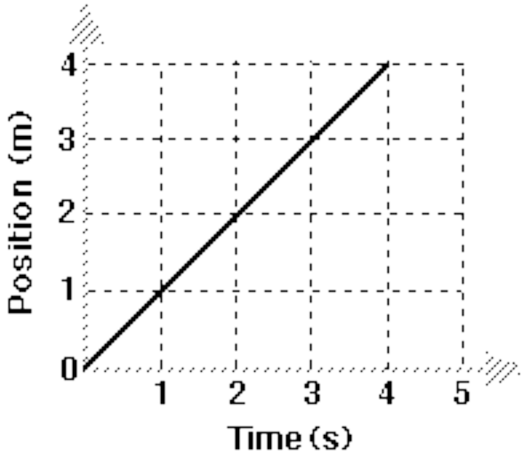
Motion Maps and Velocity vs. Time Graphs

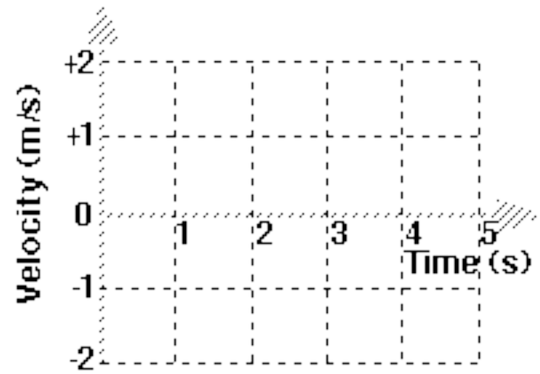
Sketch **velocity vs. time** graphs and **motion maps** corresponding to the following descriptions of the motion of an object.

1. The object is moving in the positive direction at a constant (steady) speed. Motion Map: 0 m 	
2. The object is standing still. Motion Map: 0 m 	
3. The object moves in the negative direction at a steady speed for 10s, then stands still for 10s. Motion Map: 0 m 	
4. The object moves in the positive direction at a steady speed for 10s, reverses direction and moves back toward the negative direction at the same speed. Motion Map: 0 m 	

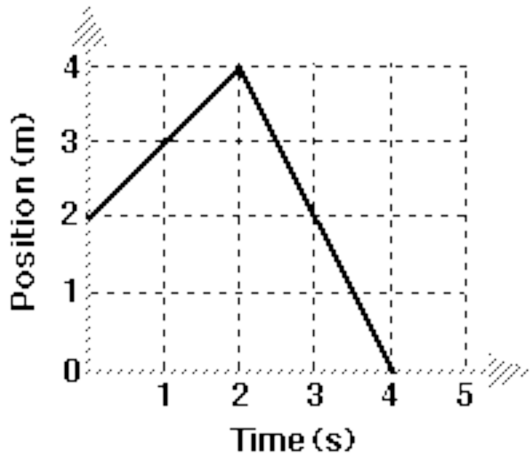
Draw the velocity vs time graphs for an object whose motion produced the position vs time graphs shown below at left.

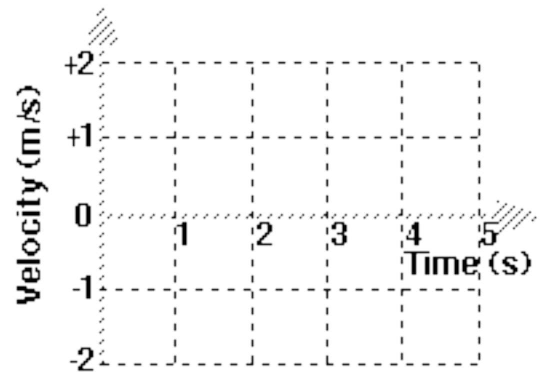
5.



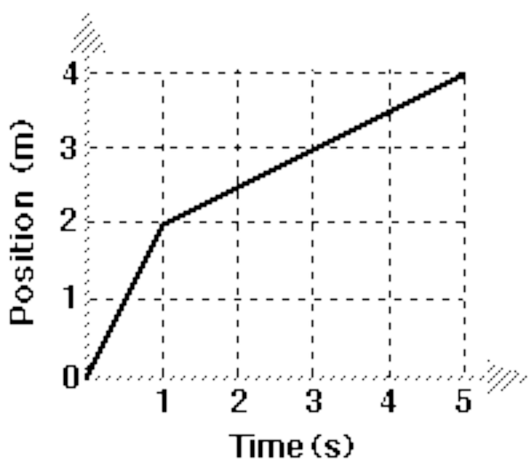


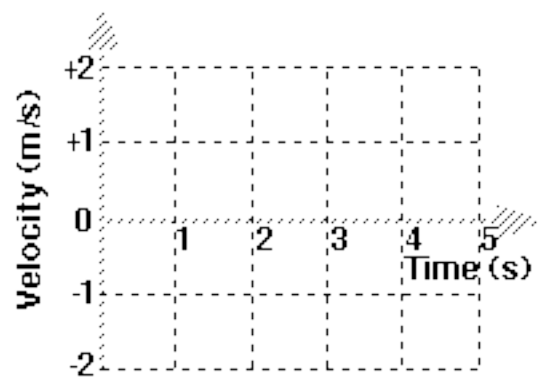
6.





7.





8. For many graphs, both the **slope** of the line and the **area** between the line and the horizontal axis have physical meanings.

a. What does the slope of a position time graph tell you about the motion of an object?

b. Looking at the velocity time graphs, determine the units for a square of area on the graph.

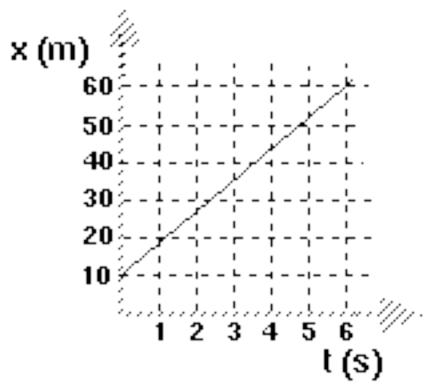
c. What does the area under the velocity-time graph tell you about the motion of an object?

Name________________________________

Date__________ Pd____

Constant Velocity Particle Model:
Review Sheet

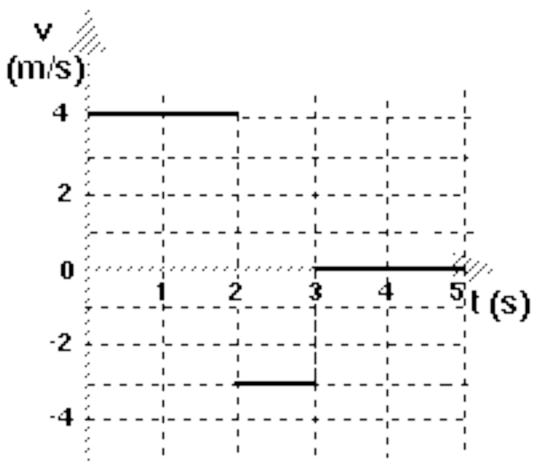
1. Consider the following position vs. time graph.



a. Determine the average velocity of the object.

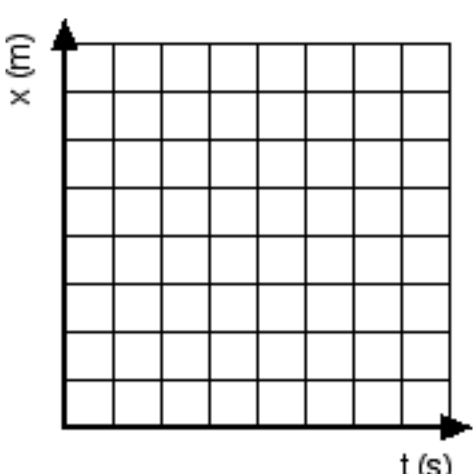
b. Write a mathematical model to describe the motion of the object.

2. Shown below is a velocity vs. time graph for an object.



a. Describe the motion of the object.

b. Draw a corresponding position vs. time graph. Number the axes. You may assume the object starts from the origin.



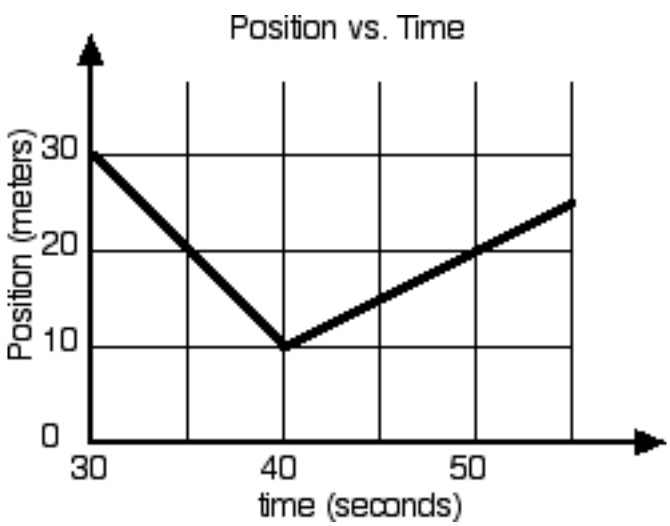
c. How far did the object travel in the interval t =1s to t =2s?

d. Find the displacement from t = 0s to t = 5 s. Explain how you got your answer.

e. Find the average velocity from t = 0s to t = 5 s. Explain how you got your answer.

f. Find the average speed from t = 0s to t = 5 s. Explain how you got your answer.

3. A bird travels toward the origin, then suddenly reverses direction.



a. Find the average velocity from t = 30s to t = 40s.

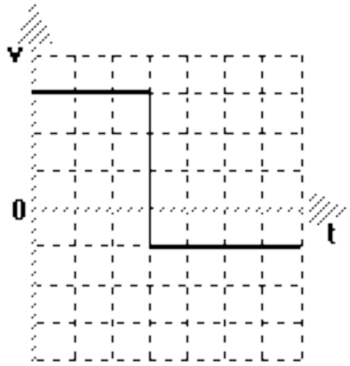
b. Find the average velocity from t = 40s to t = 50s.

c. Determine the average speed from t = 30s to t = 50s.

d. Determine the average velocity from t = 30s to t = 50s.

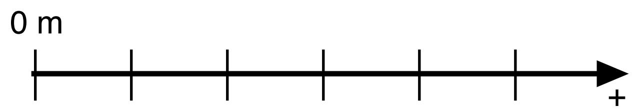
e. Find the velocity at t = 35 seconds.

4. A basketball initially travels at 3 meters per second for 3 seconds:



a. Describe the motion of the ball after t = 3 seconds.

b. Draw a quantitative motion map that models the motion of the object.



c. How far did the ball travel from t = 3s to t = 7s?

5. A racecar reaches a speed of 95 m/s after it is 450 meters past the starting line. If the car travels at a constant speed of 95 m/s for the next 12.5 s, how far will the car be from the starting line? Use the appropriate mathematical model and show how units cancel.